A Brief Introduction To Interferometry

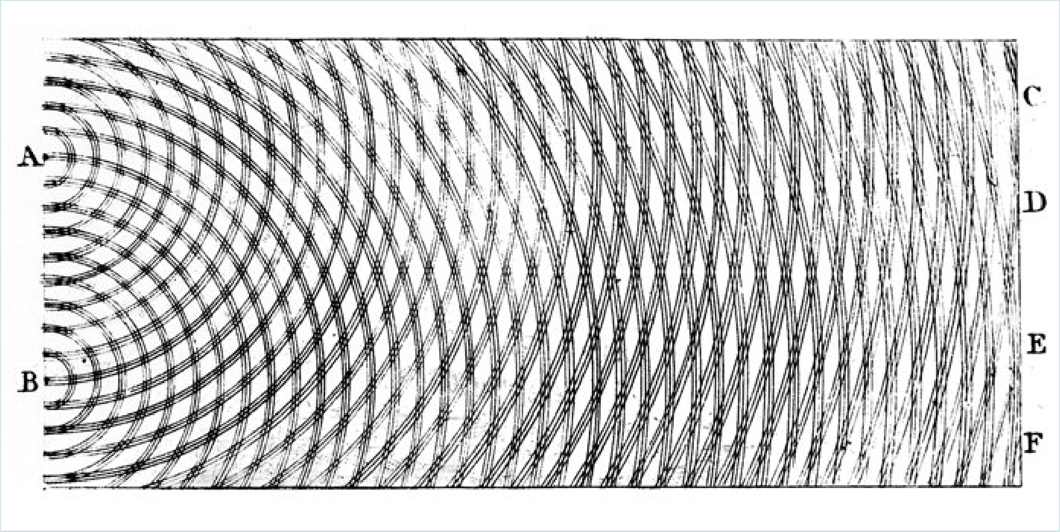
Fundamentals of Radio Interferometry

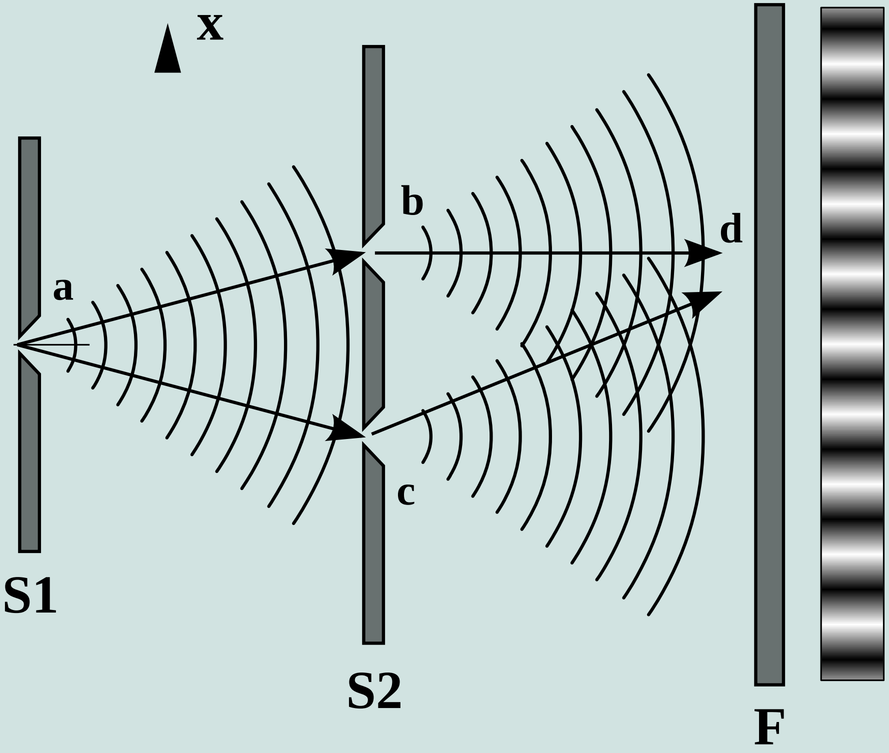
Oleg Smirnov
Rhodes University / SKA SA

NASSP 2016

The Double-Slit Experiment

- Thomas Young (1773-1829)
- Performed experiment in 1803 to demonstrate the wave-like nature of light (still controversial at the time, cf. Newton's corpuscular theory)
- Light source (S1: pinhole -- sunlight was used) is used to illuminates a screen with two slits (S2 b,c)
- ***Fringe pattern*** forms on screen F due to the waves from b and c interfering





Brightness, Phase Delay, Interference

- Light is an electromagnetic (EM) wave, described by a complex number* (*amplitude* and *phase*):

$$E = a\mathrm{e}^{\imath\phi}$$

(*: actually, two numbers – see lecture on polarisation later)

- We *measure* (eye sees, photographic plate records) its ***brightness*** or ***intensity...***

  ...averaged over time

$$B = \langle |E|^2 \rangle = \langle E E^* \rangle = a^2$$

- The same wave travelling along two different paths experiences a *phase delay*:

$$E_1 = a_1 \mathrm{e}^{\imath\phi}, \quad E_2 = a_2 \mathrm{e}^{\imath(\phi+\phi_0)}, \quad \phi_0 = 2\pi\tau_0/\lambda$$

- Two arriving waves superimpose:

$$E = E_1 + E_2$$

- The resulting brightness is:

$$(E_1 + E_2)(E_1^* + E_2^*) = |E_1|^2 + |E_2|^2 + E_1 E_2^* + (E_1 E_2^*)^*$$

$$= a_1^2 + a_2^2 + 2a_1 a_2 \cos\phi_0$$

A Toy Double-Slit Experiment

- The fringe pattern on the screen is caused by the interfering term φ_0, which varies with pathlength difference τ_0, which varies with position on the screen

- Refer to ipython notebook (1.9.2, 1.9.3) for an interactive implementation of this experiment in Python

- Note how sensitive the fringe pattern is to ***baseline*** (distance between the slits) and ***wavelength***

Interferometry: Measuring Stuff?

- Mental experiment: imagine we have built a working double-slit setup. Can we turn the experiment around, and use it as a *measurement* device? Could we measure some properties of the light source?

- Source intensity: not very interesting (don't need an interferometer)

- We can measure source position (refer to ipython notebook, 1.9.4.1)

- *Offset* in the source position results in a *shift* of the fringe pattern

- Longer baselines (or shorter wavelengths) = more accurate measurement

- Position measurement is ambiguous due to the periodic nature of the fringe pattern

- Measuring with different baselines resolves ambiguity

Coherent vs *Incoherent*

- Consider two independent light sources *a* and *b*, can their radiation interfere and form a fringe pattern?

$$E_x = a_x \mathrm{e}^{\imath \phi_x}, \quad E_y = a_y \mathrm{e}^{\imath \phi_y}$$

- Generally, no, because the sum is...

$$(E_x + E_y)(E_x^* + E_y^*) = a_x^2 + a_y^2 + E_x E_y^* + E_x^* E_y$$

- ...and once we take the average in time:

$$\langle E_x E_y^* \rangle = a_x a_y \langle \mathrm{e}^{\imath(\phi_x - \phi_y)} \rangle \to 0$$

- ...the phase difference for two independent sources is essentially random, thus the interfering terms average out to 0

- E_x and E_y are called *mutually incoherent.*

Coherent vs Incoherent

- Contrast that to radiation from one source, two paths, which is ***coherent****:

$$E_1 = a_1 \mathrm{e}^{\imath\phi}, \quad E_2 = a_2 \mathrm{e}^{\imath(\phi+\phi_0)}$$

$$\langle E_1 E_2^* \rangle = a_1 a_2 \langle \mathrm{e}^{-\imath\phi_0} \rangle$$

- ...the phase difference is constant, and therefore the interfering term does *not* average to 0

*) To an extent, vis., *coherence time:*

$$\langle E(t), E^*(t - t_0) \rangle = ?$$

Our signal is essentially noise!

Adding Up Fringe Patterns

- Radiation from one source received along two paths is coherent:

$$E_1 = a_1 e^{\imath\phi}, \quad E_2 = a_2 e^{\imath(\phi+\phi_0)}, \quad \langle E_1 E_2^* \rangle = a_1 a_2 \langle e^{-\imath\phi_0} \rangle$$

- Radiation from two sources in mutually incoherent:

$$\langle E_x E_y^* \rangle = 0$$

- For two sources illuminating two slits, we then have:

$$\langle (E_{1x} + E_{2x} + E_{1y} + E_{2y})(E_{1x}^* + E_{2x}^* + E_{1y}^* + E_{2y}^*) \rangle$$

$$= \langle |E_{1x}|^2 \rangle + \dots + \langle E_{1x} E_{2x}^* \rangle + \dots + \langle E_{1x} E_{1y}^* \rangle + \dots$$

Constant terms

Interfering terms

Incoherent terms (=0)

- Therefore, the fringe pattern from two sources is the **sum** of the fringe patterns from each individual source

Source Extent

- See notebook, 1.9.4.2
- The fringe patterns from two sources add up and can “wash” each other out, if the spacing is just right
- We can consider an extended source as a combination of many small independent “subsources”
- The fringes from all these “subsources” will tends to “wash out”, more so as we increase the source extent
- Therefore, we can measure the source extent by a reduction in the ***amplitude*** of the fringe pattern
- Historically, this was the first application of interferometry in astronomy

Visibility

- The term ***visibility*** was originally introduced to describe the contrast (or amplitude) of the fringe pattern, as $V=(max-min)/(max+min)$
  (a very literal term: in early experiments fringes were measured “by eye”)

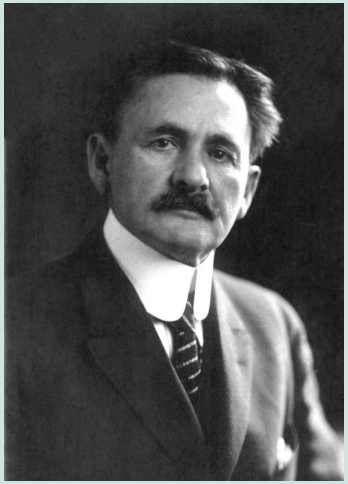
- Radio interferometry deals in *complex visibilities:*
  - the amplitude of the fringe corresponds to the visibility amplitude
  - the phase (i.e. offset) of the fringe corresponds to the phase

- <u>**All the information in the fringe pattern can be encoded in that one complex number**</u>

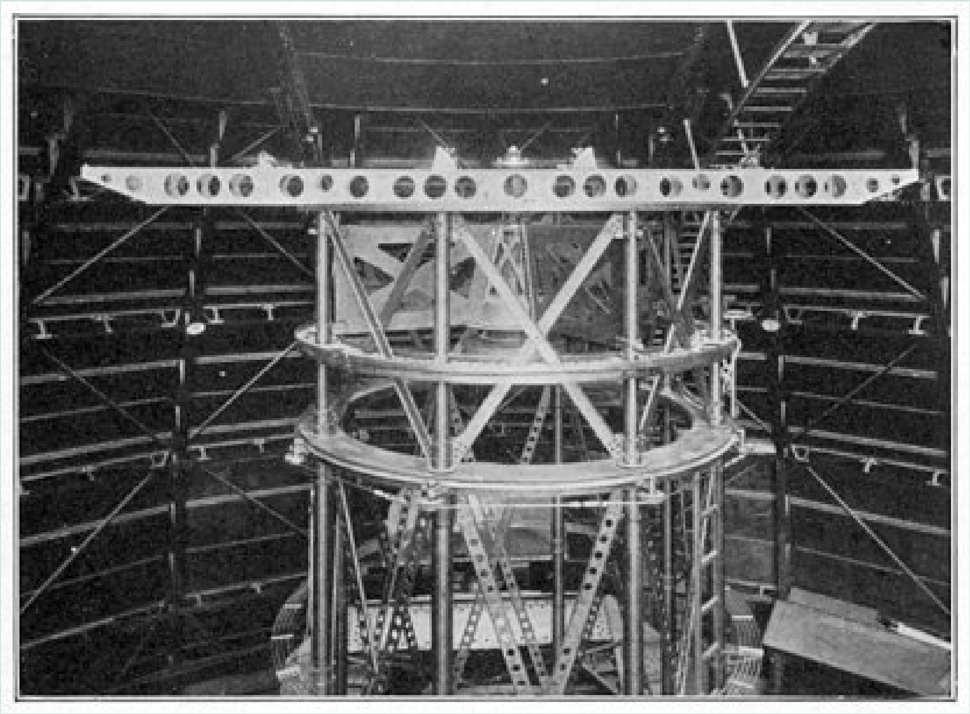
- Visibility amplitude encodes source shape, visibility phase encodes source position (we will return to this in Chapter 4)

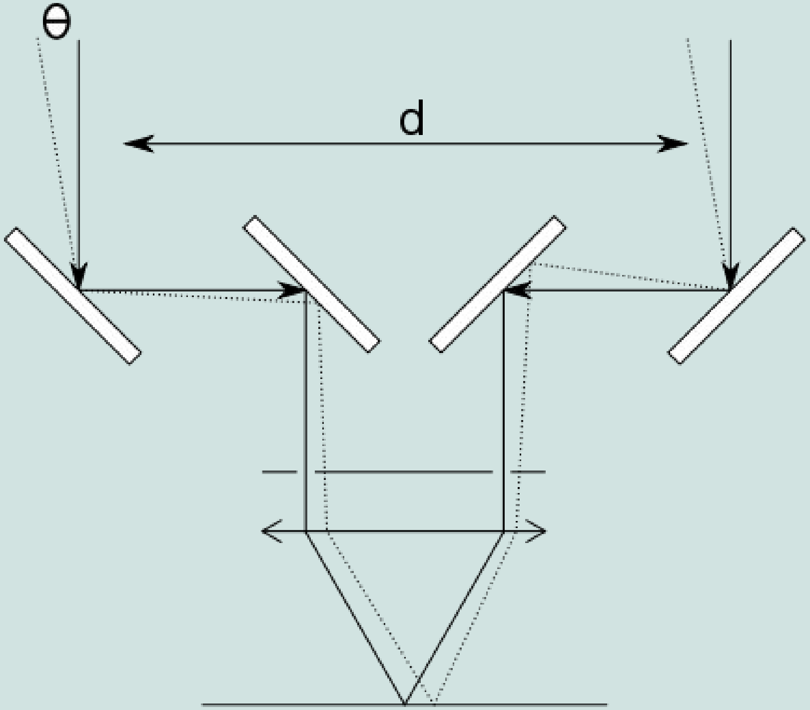
- For technical reasons, phases are much harder to measure accurately, thus all early interferometric experiments dealt in amplitudes

The Michelson Stellar Interferometer



- The first interferometric experiment in astronomy (Michelson & Pease 1921, Astrophys. J. **53**, 249–259)
- Constructed at Mount Wilson Observatory
- Fringes observed through an eyepiece!





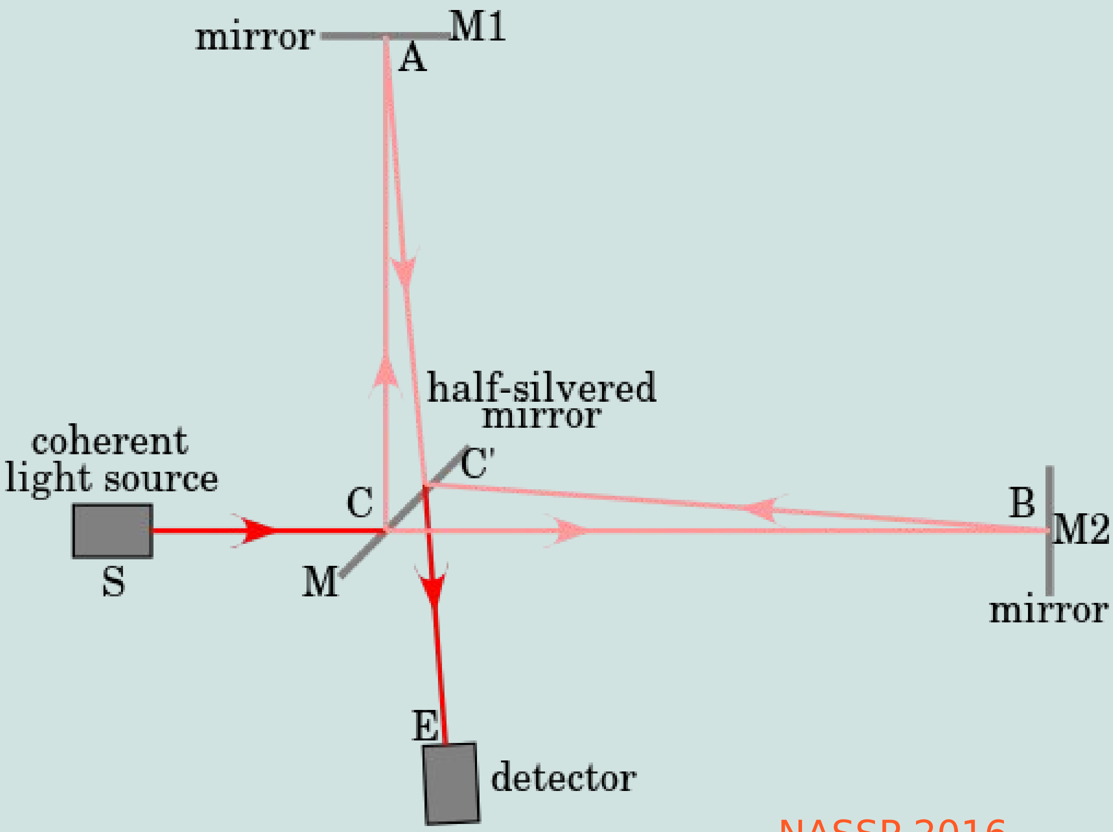
The Betelgeuse Size Measurement

- Michelson & Pease used the Mount Wilson interferometer to measure the size of the red giant Betelgeuse
- Experimental setup: adjustable baseline up to ~6m
- Started at 1m, and observed fringes from Betelgeuse (after a lot of fiddling...)
- Adjusted baseline up in small increments and observed the fringe visibility decrease, until at ~3m they could no longer see fringes
- From this, inferred the size of Betelgeuse to be ~0.05 arcsec
- See notebook appendix for a toy recreation of this

Other Famous Interferometric Experiments

- What else can we measure with an interferometer? Observe that the fringes are extremely sensitive to the geometry of the instrument itself
  - ...and wavelength (Thomas Young)

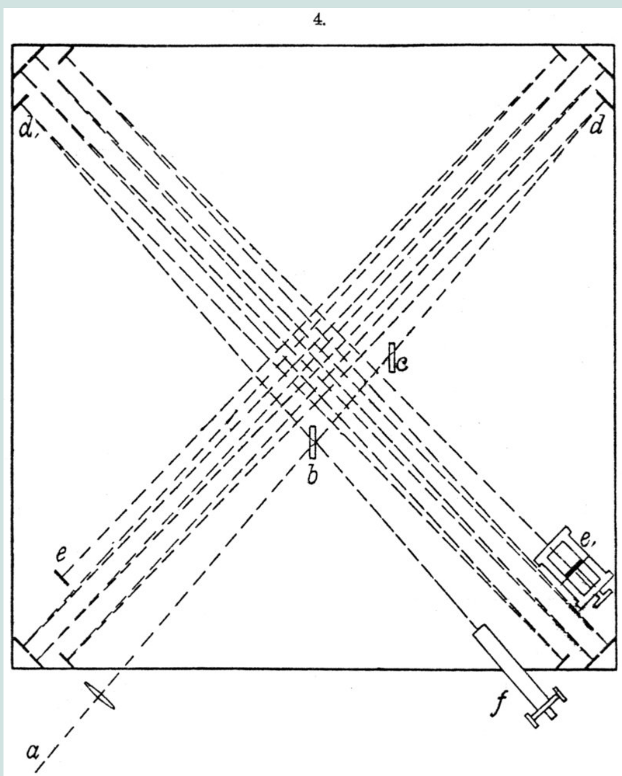
- We can design careful experiments to measure changes in geometry

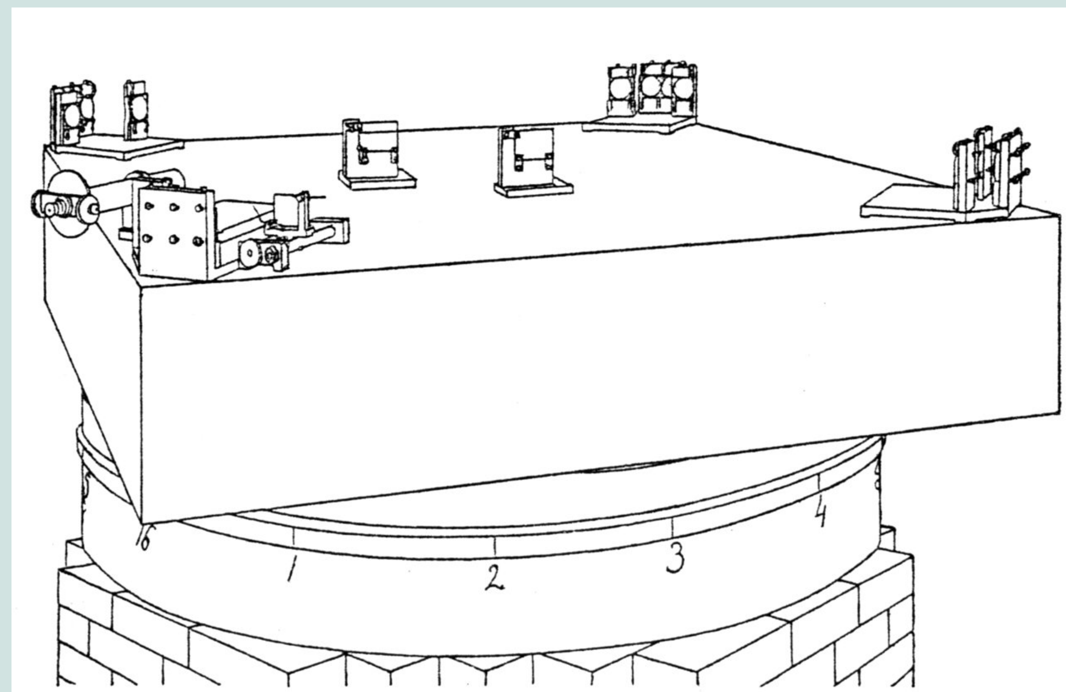


Michelson-Morley (1887)

- Attempted to measure the relative motion of matter through the "luminous aether"

- Negative measurement undermined the aether theory and eventually led to special relativity

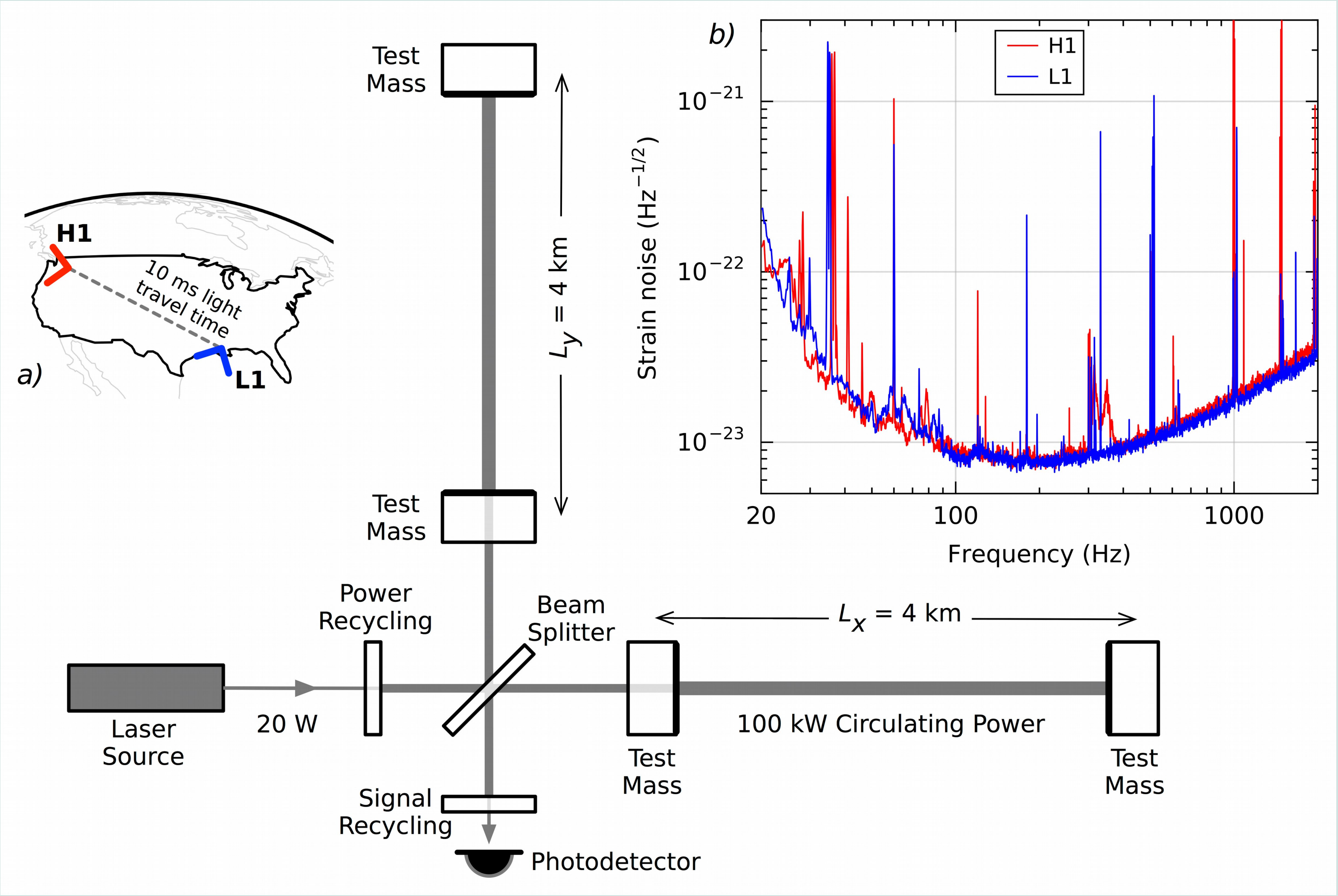




- Ring any bells?

LIGO (2015)

- Interferometric measurement of gravitational waves



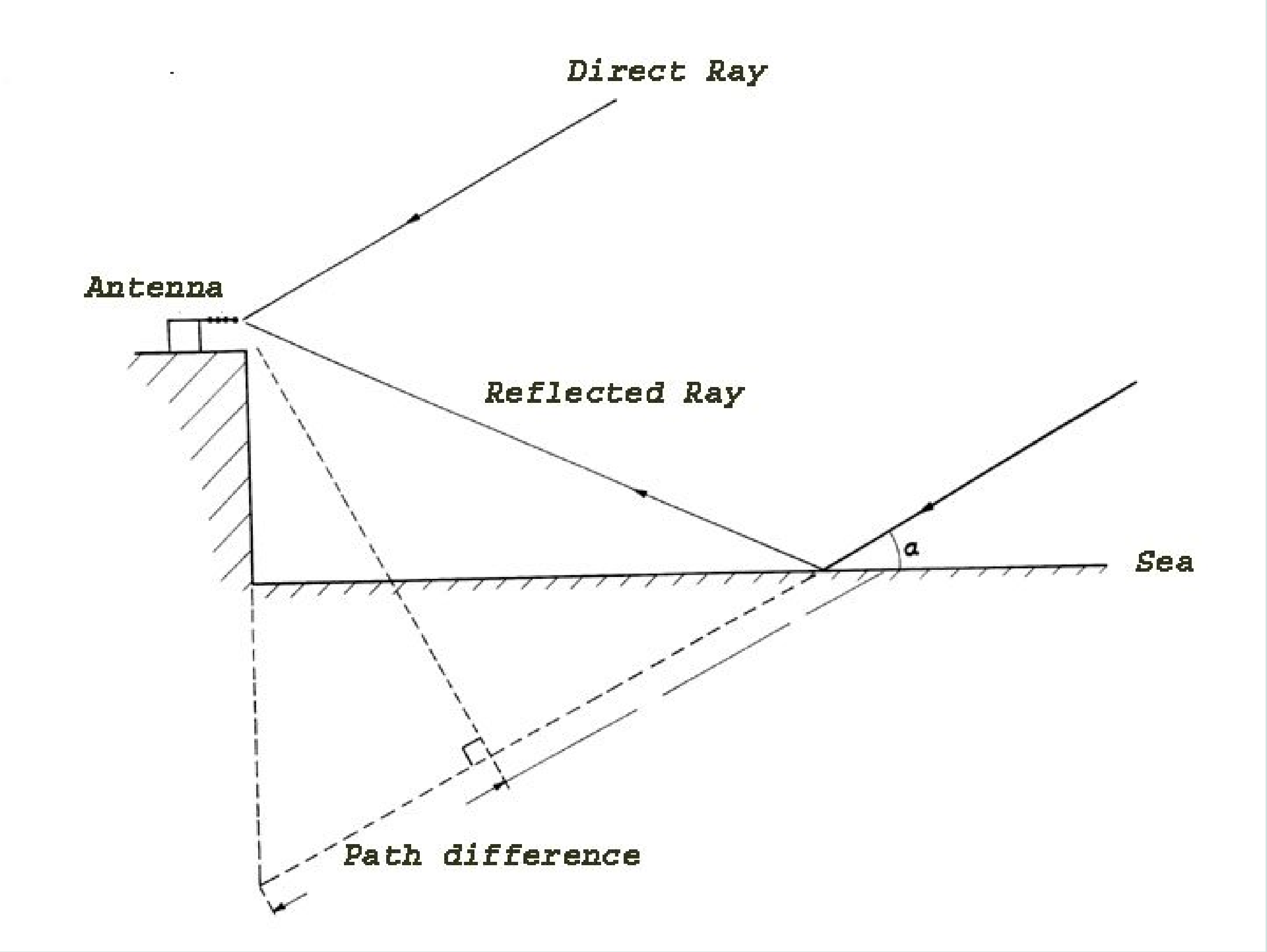
Modern Optical Interferometry

- Modern optical interferometers still follow the basic Michelson design
- Example: the Very Large telescope (ESO)

Early Radio Interferometry

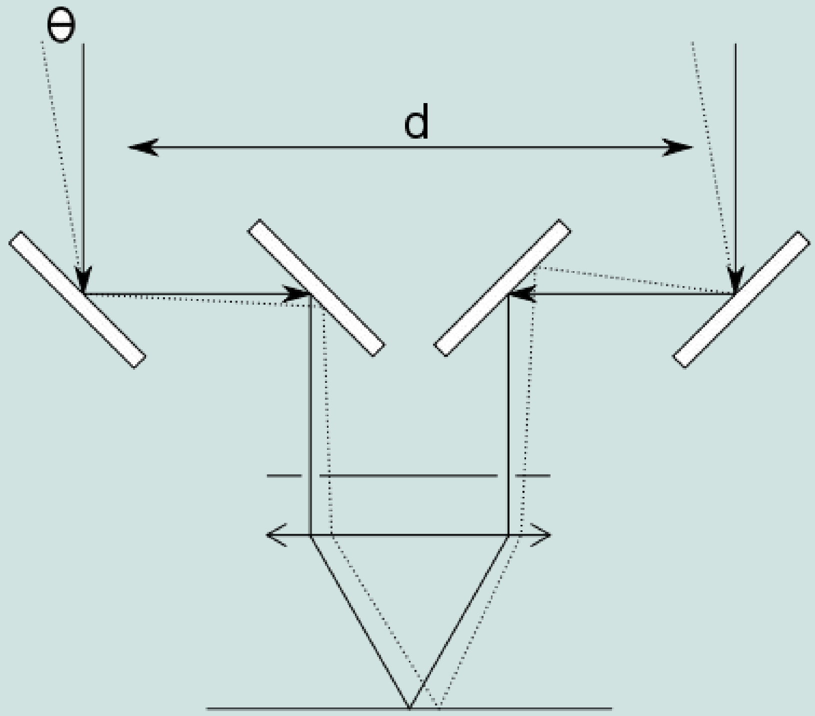
- Sea-cliff interferometer (Australia 1945-48)

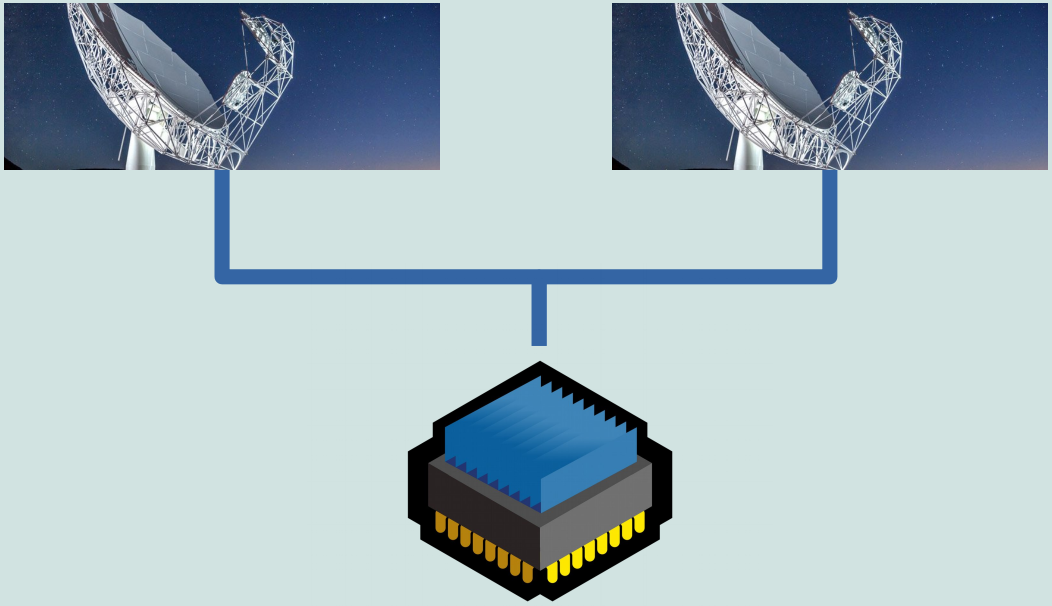
- Measure the sum of two signals by a single antenna:



Modern Radio Interferometry: Going Digital

- Radio telescopes can directly sample the incoming EM front
- Replace the optical "light path" by electronics:





Additive vs. Multiplicative Interferometers

- Optical interferometers (and e.g. the sea cliff interferometer) are additive, since they measure

$$\langle |E_1 + E_2|^2 \rangle$$

- Recall that

$$|E_1 + E_2|^2 = |E_1|^2 + |E_2|^2 + E_1 E_2^* + (E_1 E_2^*)^*$$

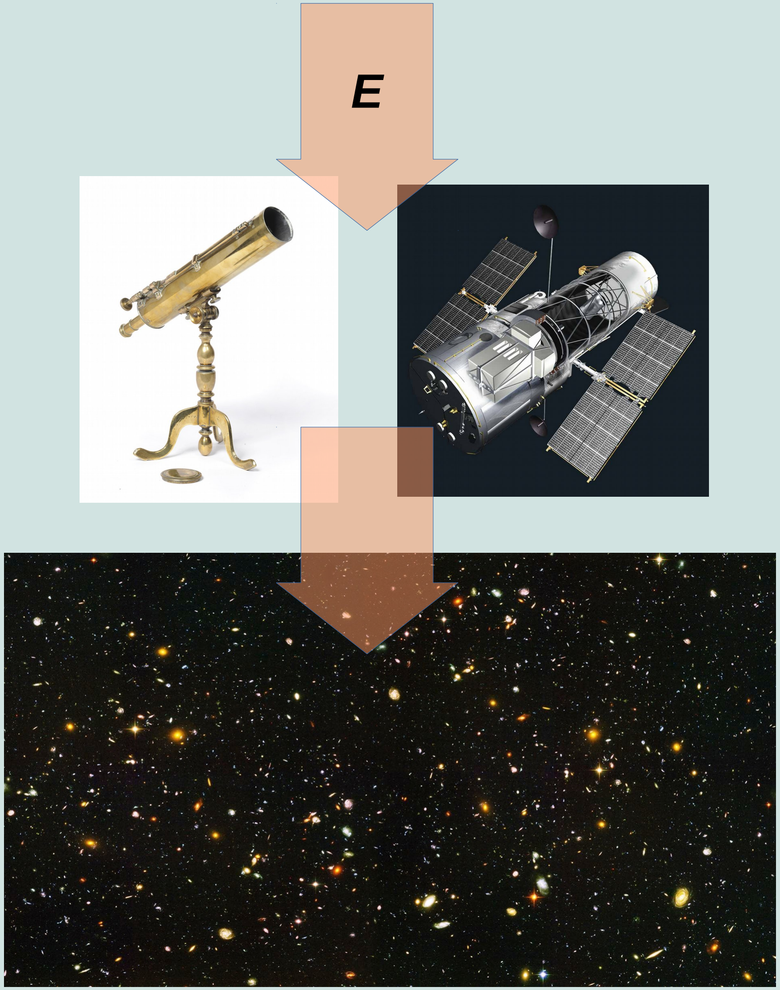
  - ...and that all the interesting “interferometric” information is in the cross-terms

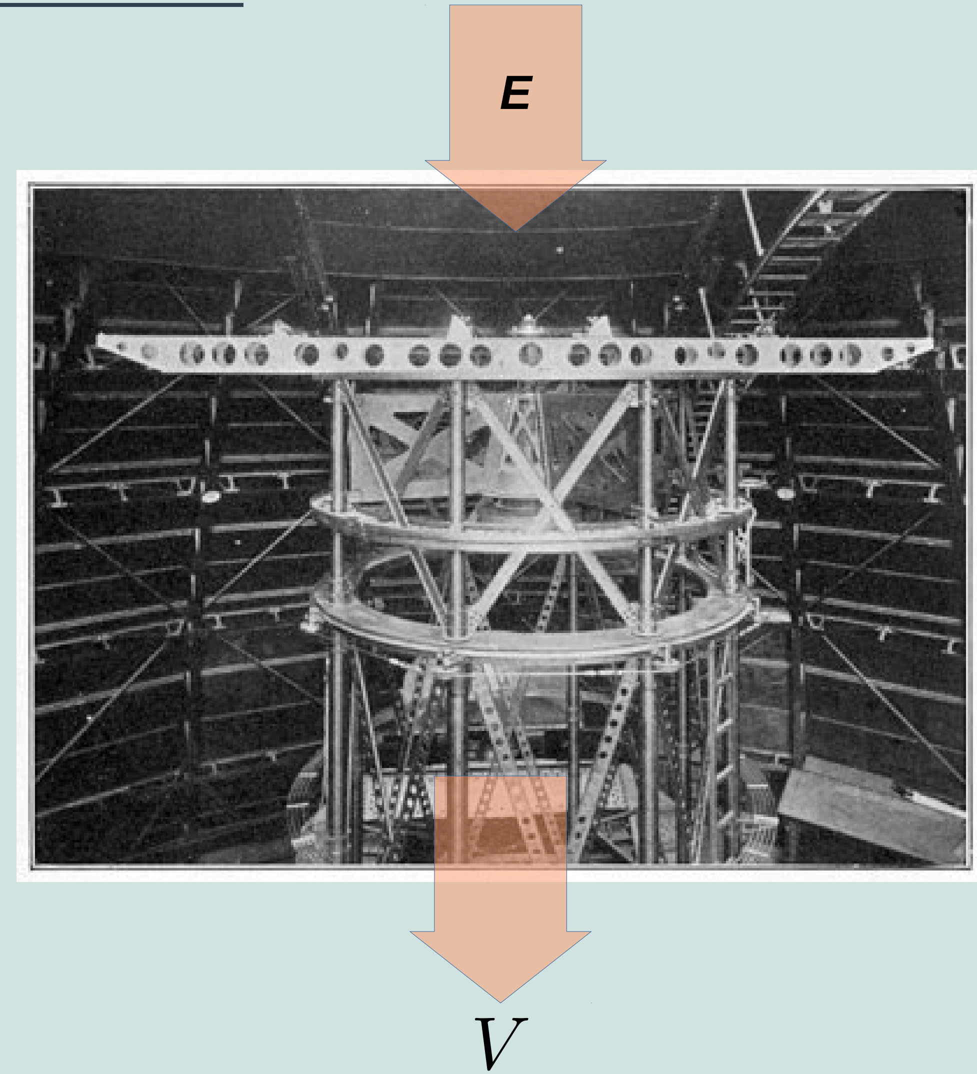
- Modern radio interferometers are *multiplicative*, directly computing

$$\langle E_1 E_2^* \rangle$$

- This is only possible in the radio regime, where we can directly sample and record the signals $E_1, \ E_2$

Imaging Vs Single Measurement





- A traditional telescope produces an image (=lots of data)
- Early interferometers would produce a single number

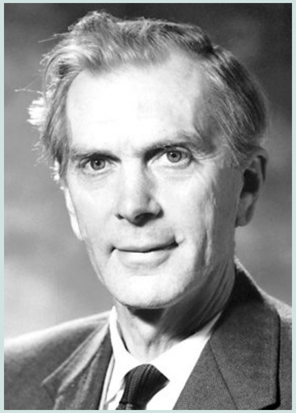
Developing Aperture Synthesis

- As we saw earlier (see notebook 1.9.6), a single visibility conveys some information about the source structure
  - ...with ambiguity

- Multiple visibilities (on different baselines) provide additional information

- With sufficient visibility measurements, one can reconstruct an image of the sky
  - ...each visibility measures a *Fourier mode* of the sky brightness distribution

- Sir Martin Ryle: 1974 Nobel Prize for development of this ***aperture synthesis*** technique



- Modern radio interferometry is aperture synthesis

Aperture Synthesis

- With the development of aperture synthesis, radio interferometers have become true imaging devices, with resolution far in excess of that available to optical telescopes

JVLA image of a galaxy cluster, 2-4 GHz, <1” resolution (data courtesy E. Murphy)

Conclusion

- Modern radio interferometry uses arrays of radio telescopes to image the sky via the *aperture synthesis* technique

- This is what this course is all about